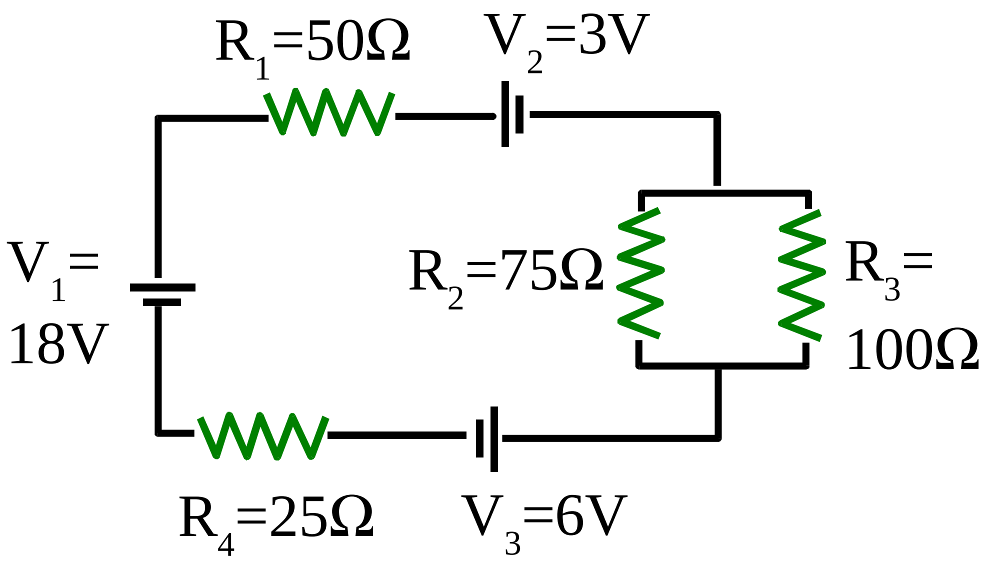
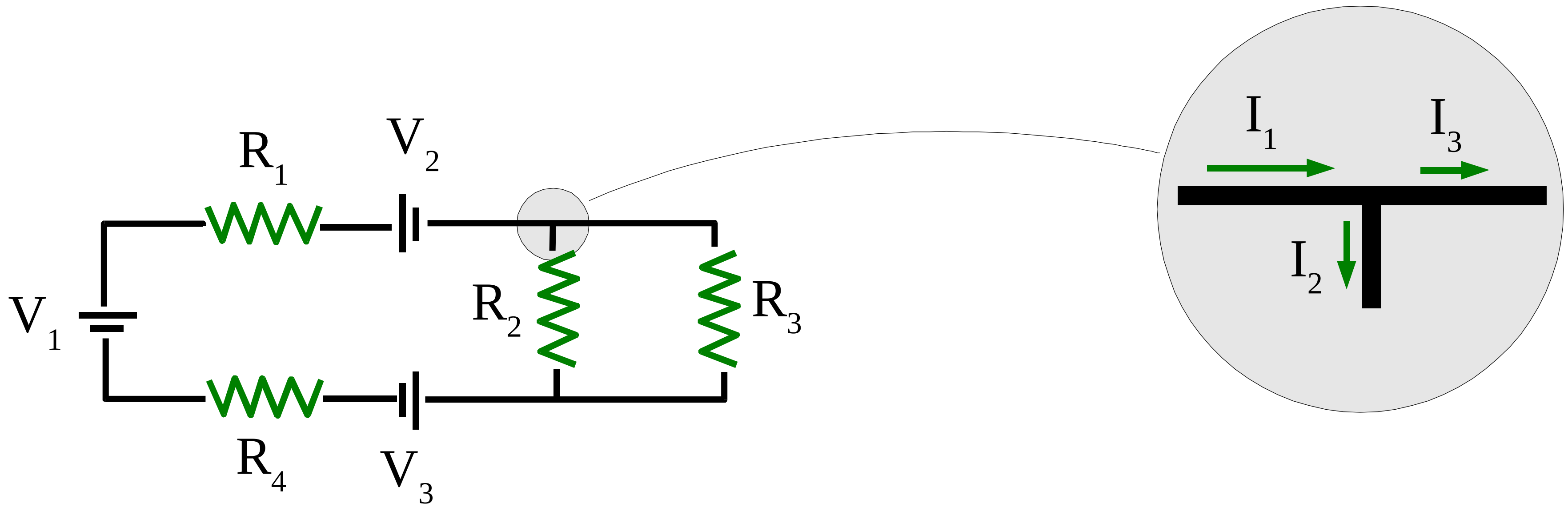
More Complicated Circuits



What happens if you have a much more complicated circuit? The circuit has 3 sources of emf (3 batteries), R_2 and R_3 are two resistors in parallel, and resistors R_1, R_{23eq}, and R_4 are in series.

To solve this, we need to first define the direction of current from each battery. We choose the current to flow from the positive side of the battery and into the negative side. This *convention* dates from the original belief that the current in metals was made from positive charges. We now know it is electrons (negative charges) which flow readily in metals.

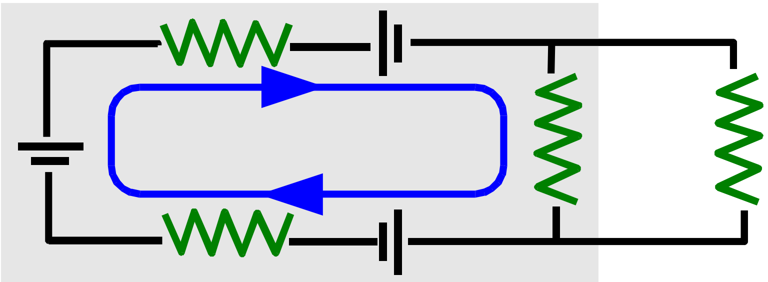
Current Junctions

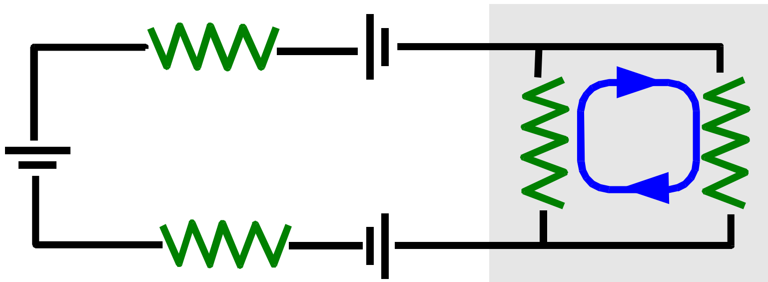


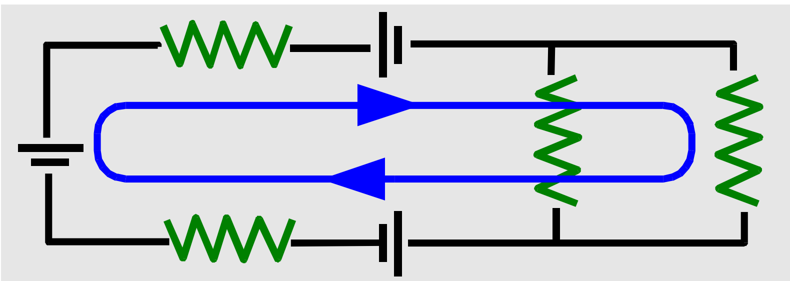
Junctions deal with the current at a point where it can take two different paths. We use conservation of charge to explain this.

This allows one to write one equation for each junction. In this case, the equation is: $I_1 = I_2 + I_3$

Current Loops







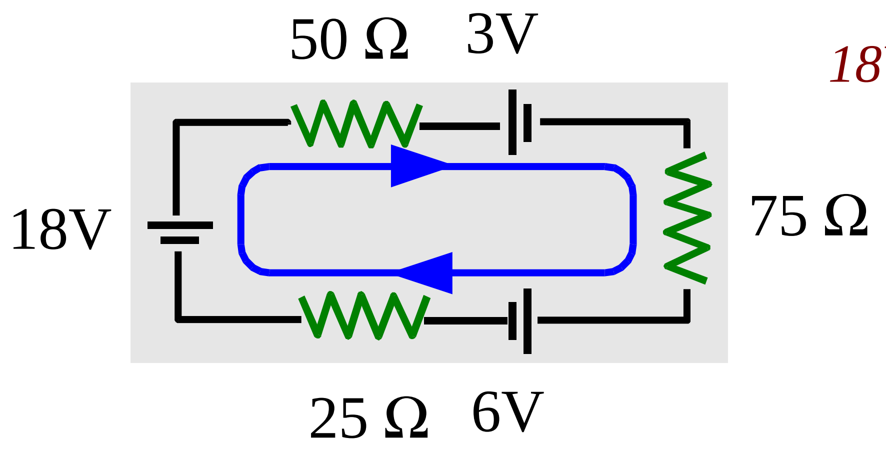
We choose to go around each loop in a clockwise direction. This is an arbitrary choice and the physics would not change if you go the other direction.

For each loop:

$$\Sigma\, Emf = \Sigma\, \mathrm{I\ R's}$$

You need to pay attention how you cross a battery and also the direction you chose for the current when you declared your current junction equation.

Loop 1 and Loop 2

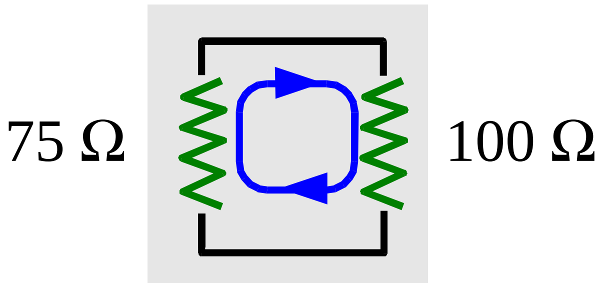


$$\Sigma\, Emf = \Sigma\, I\, R's$$

$$18V - 3V - 6V = I_1(50\Omega) + I_2(75\Omega) + I_1(25\Omega)$$

$$9 = (50+25)I_1 + 75I_2$$

$$9 = 75I_1 + 75I_2$$



$$\Sigma\, Emf = \Sigma\, I\, R's$$

$$0V = I_3(100\Omega) - I_2(75\Omega)$$

$$100I_3 = 75I_2$$

$$I_3 = \frac{75}{100}I_2$$

$$I_3 = \frac{3}{4}I_2$$

Solve the Problem

We could solve for loop three as well but we do not need it. We have three equations and only three unknowns:

$$(A) \quad I_1 = I_2 + I_3$$

$$(B) \quad 9 = 75 I_1 + 75 I_2$$

$$(C) \quad I_3 = \frac{3}{4} I_2$$

(C) *into* (A)

$$I_1 = I_2 + \frac{3}{4} I_2$$

$$(D) \quad I_1 = 1.75 I_2$$

(D) *into* (B)

$$9 = 75(1.75 I_2) + 75 I_2$$

$$9 = (131.25 + 75.00) I_2$$

$$(E) \quad I_2 = 0.044\,A$$

(E) *into* (C)

$$I_3 = \frac{3}{4}(0.044)\,A$$

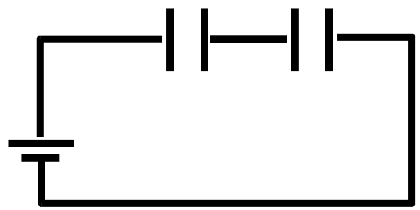
$$(F) \quad I_3 = 0.033\,A$$

(F) and (E) *into* (A)

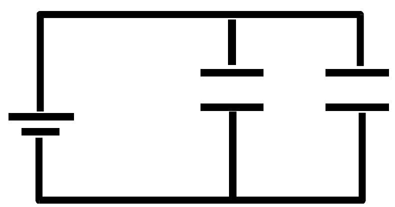
$$I_1 = 0.044\,A + 0.033\,A$$

$$I_1 = 0.077\,A$$

Capacitors in Series and Parallel Circuits



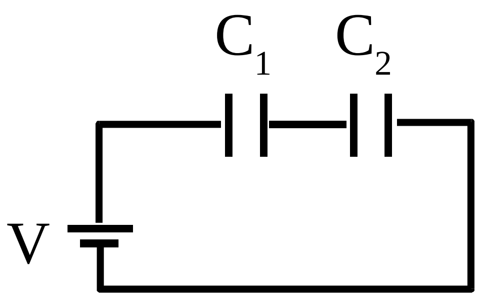
When you have capacitors in series, the charge on both sets of plates is equal. The voltage gets split between both capacitors.



When you have capacitors in parallel, the voltage across each capacitor is the same. The charge gets split between the two.

Capacitors in Series

Here, the charge on each plate is the same
so the total voltage is split between both capacitors.



$$V_T = V_1 + V_2$$

$$\text{Since, } q = CV \Rightarrow V = \frac{q}{C}$$

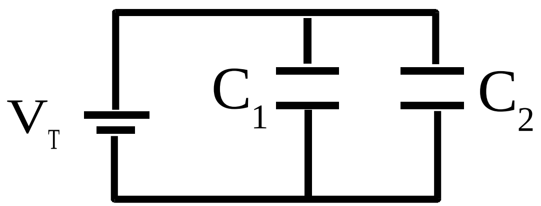
$$\frac{q}{C_T} = \frac{q}{C_1} + \frac{q}{C_2}$$

therefore

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

Notice how this is *opposite* resistors in series

Capacitors in Parallel



Here, the voltage across each capacitor is the same so the charge is distributed across both capacitors.

$$q_T = q_1 + q_2$$

Since $q = CV$

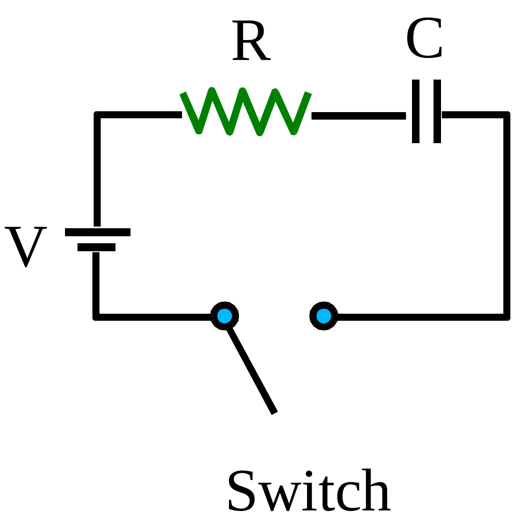
$$C_T V = C_1 V + C_2 V$$

therefore

$$C_T = C_1 + C_2$$

Notice how this is the *opposite* to resistors in parallel

Circuit with both a Resistor and a Capacitor.



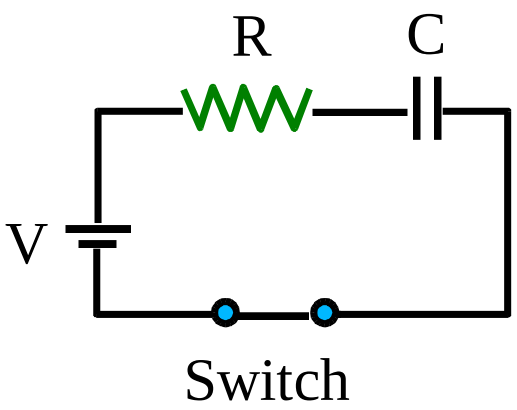
With the switch open, there is no complete circuit so charge does not flow.

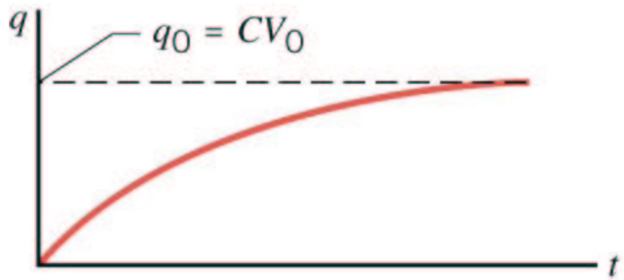
Let's say there is no charge on the capacitor originally. Once the switch is closed, charge flows from the one capacitor plate, across the switch, through the battery, and through the resistor before it is deposited on the second capacitor plate.

Remember that I (current) is how much charge per time moves through a point in the circuit. Also recall that I = V/R (Ohm's Law) so we may write:

$$I = \frac{V}{R} \Rightarrow \frac{\Delta q}{\Delta t} = \frac{V}{R} \Rightarrow \Delta q = \frac{V}{R}\Delta t$$

Capacitor Charging





Cutnell & Johnson
Wiley Publishing
Physics 5th Ed.
Figure 20.38 (W784)
C M Y K

$$\Delta q = \frac{V}{R}\Delta t$$

but, for any capacitor

$$q = CV \Rightarrow V = \frac{q}{C}$$

$$\Delta q = \frac{q}{RC}\Delta t$$

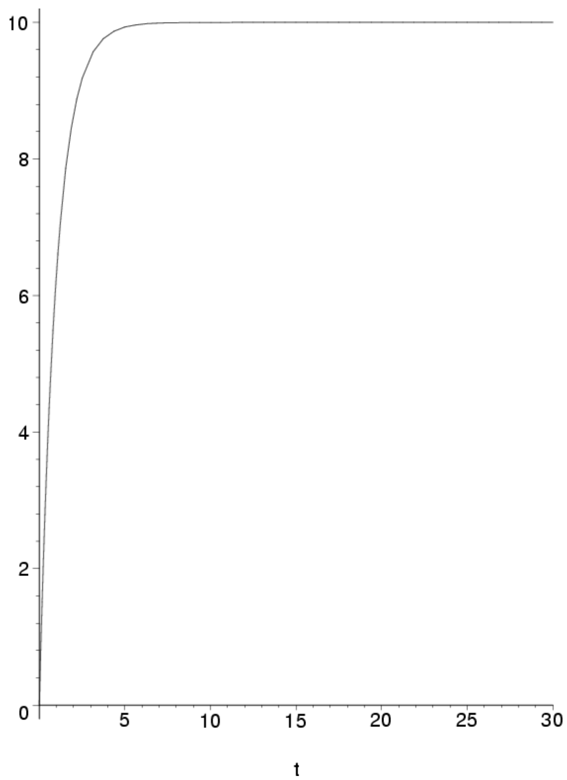
Solving how the amount of charge
on the capacitor plates
changes with time, we find (using calculus):

$$q = q_0\left[1 - e^{-t/(RC)}\right]$$

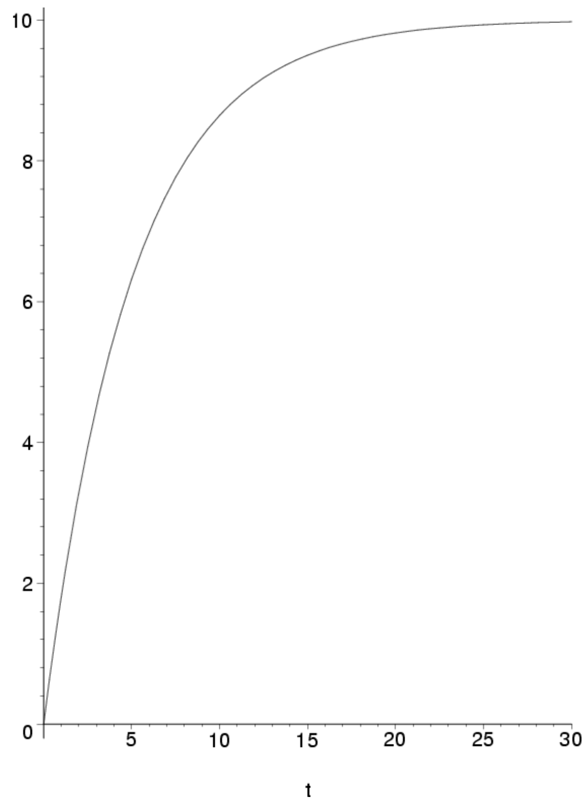
RC is known as τ. This is the time it takes for the capacitor to become 63.2% charged. This is also known as the characteristic time of the capacitor.

Different Resistors Change Charging Time

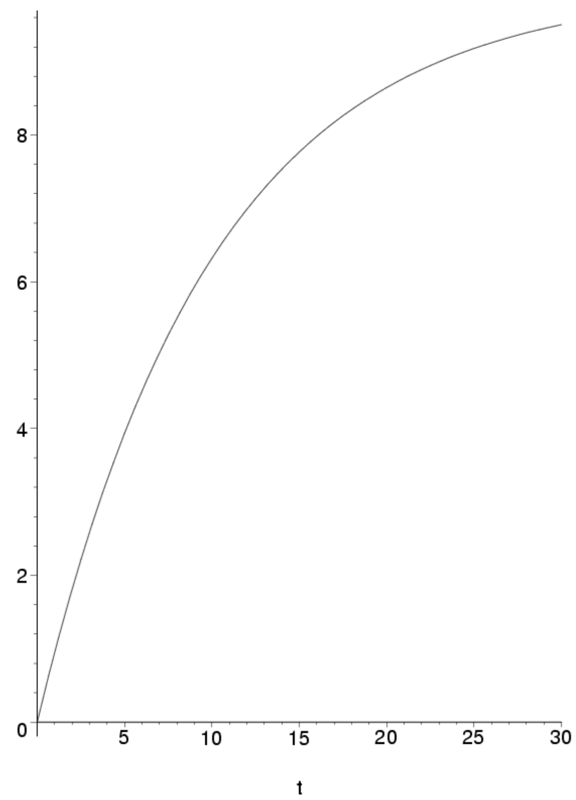
R=1



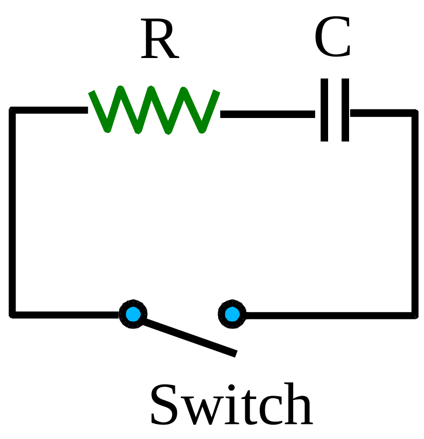
R=5



R=10

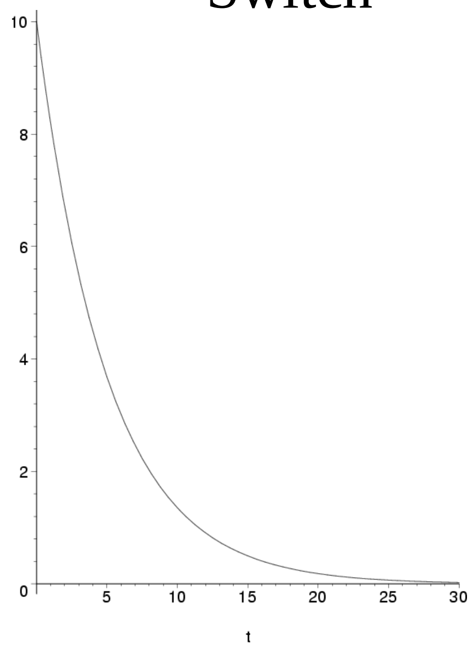


Capacitor Discharging



Here you have a case where the capacitor has stored charge and you wish to discharge it. Once the switch is closed, the situation is given by the following equation:

$$q = q_0 e^{-t/RC}$$



Here, RC or τ, is the time it takes the capacitor to lose 63.2% of its charge.