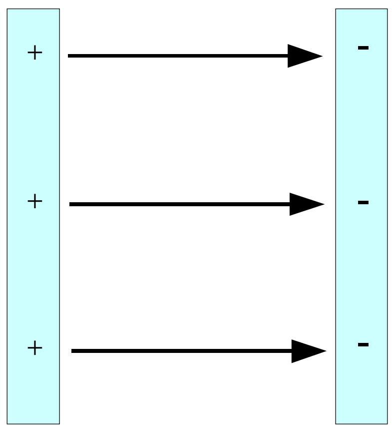
Capacitors



Start with two metal plates that each have the same but opposite charge on them.

This is called a Capacitor.

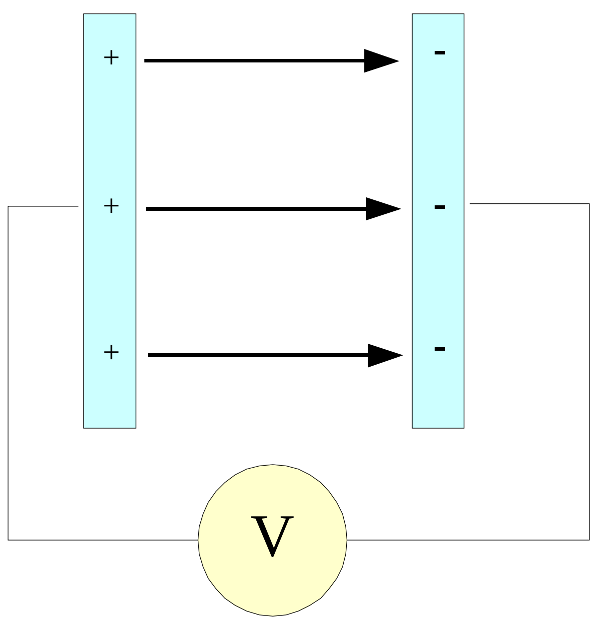
We have already seen the electric field between the two plates is given by:

$$E = \frac{\sigma}{\epsilon_0}$$

$$\text{where } \sigma = \text{charge density} \equiv \frac{q}{A}$$

$$E = \frac{q}{\epsilon_0 A}$$

Capacitance



Now hook up a voltmeter and measure the voltage between the two plates.

Recall from last lecture:

$$\Delta V = -E \Delta s$$

but we take the voltage at the one plate to be zero and set the change in voltage to be V

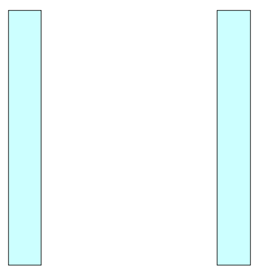
$$V = E \Delta s = \frac{q}{\epsilon_0 A} \Delta s$$

Now we have an equation relating V and q!

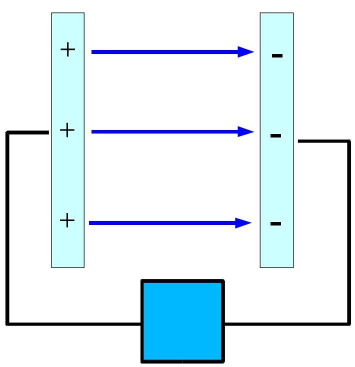
$$q = \epsilon_0 \frac{A}{d} V$$

$\epsilon_0 \frac{A}{d} \equiv$ Capacitance for a parallel plate capacitor

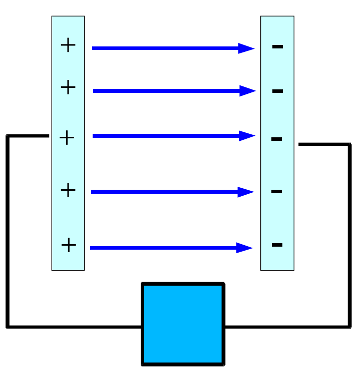
Charging a Capacitor



Now lets worry about the charging process of a capacitor. We start with two uncharged metal plates.

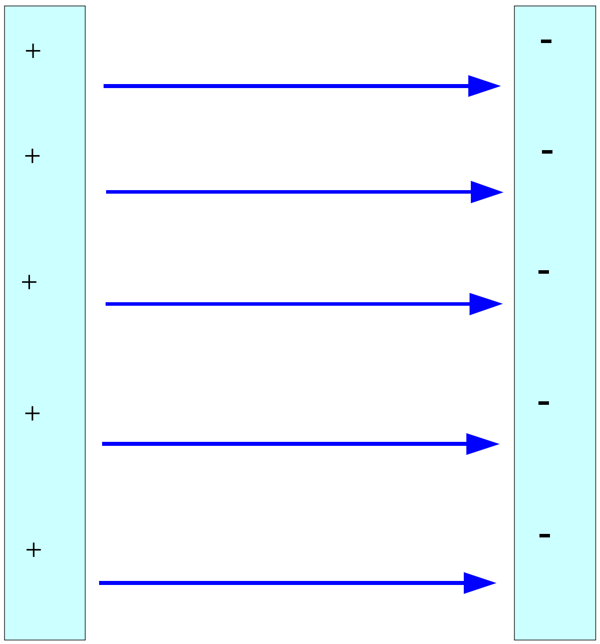


We now attach a device which is capable of moving electrons from one plate to the other. Once we have an excess of electrons on the one plate and a deficiency on the other, an electric field is produced between the two plates.



We now let our device transfer even more electrons from the one plate to the other one. As we transfer more charge, the amount of work per charge the device does in moving this charge increases! Why?

Energy Stored?

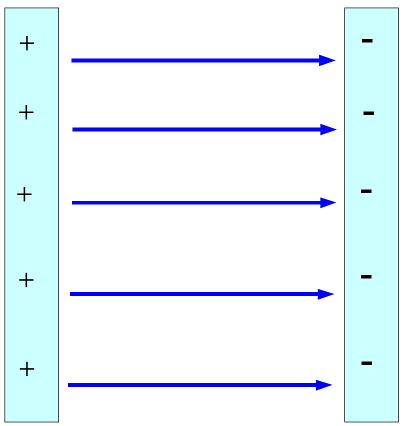


We now remove the device that moved the charges from the one plate to the other. The charges now have no way to move back to the other plate.

Remember that it took work (energy) to move the charges in the first place. Since energy is conserved, it must be stored somewhere! Where is it stored?

This form of energy storage is very common and useful! If we now connect some other device to this capacitor, we can use the stored energy to do work.

Dielectrics

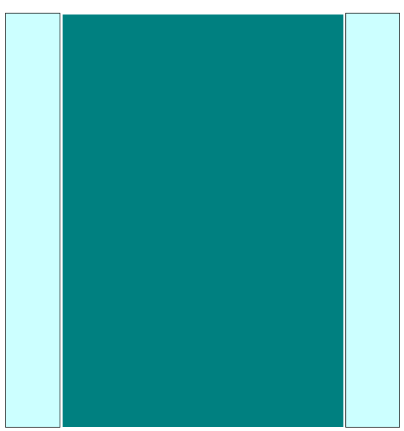


The previous slides describe a capacitor that has air or a vacuum between the two plates.

$$C = \epsilon_0 \frac{A}{d} \quad \Rightarrow A\uparrow C\uparrow ; d\downarrow C\uparrow$$

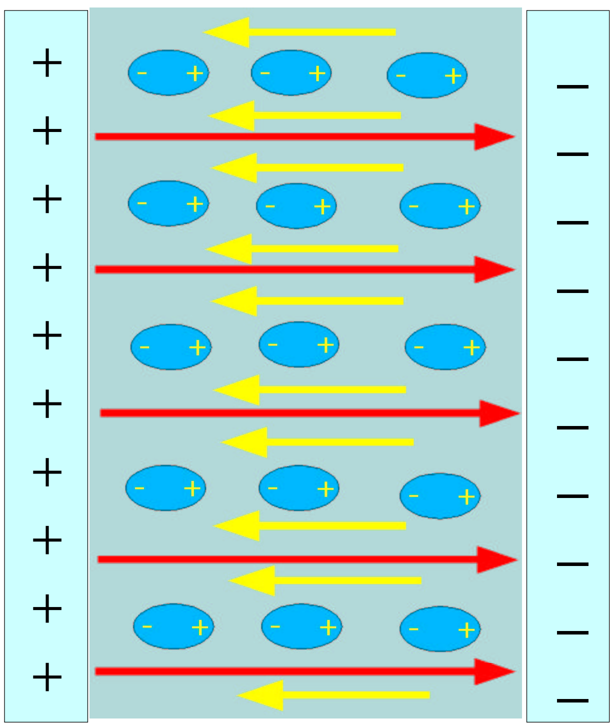
Is there a way to take a given capacitor and increase the amount of charge it can store?

Yes!



We can introduce a material known as a dielectric between the two metal plates. A dielectric material is an insulating material (no charge can easily move through it).

Dielectrics Constant



When an electric field is passed through a dielectric, the material from which the dielectric is constructed becomes polarized (the molecules in the material align with their more positive side facing the negative plate and the more negative side facing the positive plate. The individual molecules cannot move inside the material, only rotate and align with the field). This polarization of the material produces an electric field in the opposite direction of the original field. When you add the two fields together, you find a total field less than the original one.

We can now define a constant known as the dielectric constant.

Dielectric Constant

The dielectric constant is defined as:

$$\kappa = \frac{E_0}{E}$$

where E_0 is the field in the capacitor without the dielectric inserted and E is the field once the dielectric is inserted.

Table of Dielectric Constants (from the book)

Substance	κ
Vacuum	1
Air	1.00054
Teflon	2.1
Paper (royal gray)	3.3
Neoprene Rubber	6.7
Water	80.4

Capacitance with a Dielectric

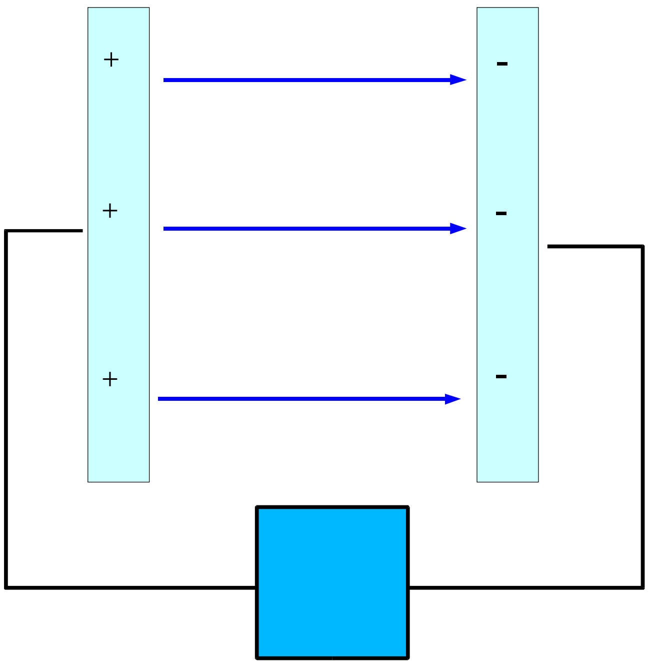
We can now rearrange things algebraically :

$$\kappa = \frac{E_0}{E} \quad \text{becomes} \quad E = \frac{E_0}{\kappa}$$

so we can write out the capacitance of a capacitor with a dielectric to be:

$$C = \frac{\kappa \epsilon_0 A}{d}$$

Energy stored in a Capacitor



We have seen the work involved in moving a charge q against a potential V is:

$$W = qV$$

As we charge a capacitor, the voltage continuously changes. When it is uncharged, there is 0 voltage between the two plates. Let's say there is a voltage V when the capacitor is fully charged. We now can find the average voltage between uncharged and fully charged.

$$V_{\text{ave}} = \frac{V_i + V_f}{2} = \frac{V}{2} \quad (\text{if } V_i = 0 \text{ and } V_f = V)$$

$$W = q\frac{V}{2} \text{ and we have seen } q = CV$$

combine them to find $W = \dfrac{CVV}{2} = \dfrac{CV^2}{2}$