Chapter 14

The Ideal Gas Law and Kinetic Theory

The Ideal Gas Law

This law deals with how a gas behaves subject to these following variables:

1) A change in pressure of the system
2) A change in temperature of the system
3) A change in volume of the system
4) A change in the number of moles of gas within the system

An ideal system is one which has certain simplifications applied to make the physics easier to understand and calculate.

In this case, we assume the individual gas particles within the system do not interact with each other. This means you don't worry about effects such as particles attracting one another etc.

Moles and Avogadro's Number

Everyday objects contain an incredible number of individual atoms or molecules. For instance, small pieces of metal may contain 10^{24} or more atoms.

It is more convenient to express the number of particles in terms of a more manageable unit. This unit is called the mole.

It was decided that one mole of anything will equal the number of atoms in 12 grams of carbon-12 (one of two isotopes of carbon).

Therefore, 1 mole = 6.022×10^{23} particles. This large number is named after Amedo Avogadro and is called Avogadro's number.

$$n = \frac{N}{N_A} = \frac{m_{\text{particle}} N}{m_{\text{particle}} N_A} = \frac{\text{total mass}}{\text{mass per mole}}$$

Atomic Mass

The mass of different materials can be very different even if the two objects made from these different masses have the same size.

An example of this would be a bar of aluminum and a similar bar of lead. For the same size bar, the lead bar would be over 7.5 times the mass (and weight) of the aluminum bar.

If one looks at a periodic table of elements, there is a listing of atomic mass which describes how many atomic mass units a given atom has.

$$1u = 1.6605 \times 10^{-27} kg$$

To find the molecular mass of a molecule, you take the individual atoms which make up the molecule, find their atomic masses, then add them all up.
Nitric Acid = NHO_3 = 1(14.0067) + 1(1.00794) + 3(15.9994) = 63.0128u

Ideal Gas Law

$$PV = nRT$$

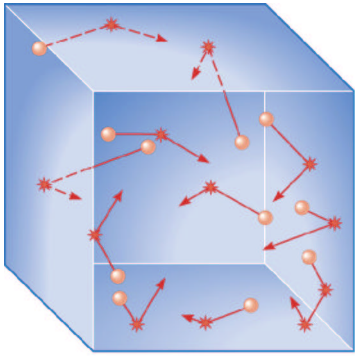
R is the universal gas constant = 8.31 J/(mol K), P is the pressure of the gas (pascals), V is the volume of the gas (m^3), T is the temperature of the gas (K) and n is the number of moles of gas there are in the system.

This simple looking law was the result of many different experimental observations.

Boyle's Law (constant n and T): $$P_i V_i = P_f V_f \rightarrow P_f = P_i \times \frac{V_i}{V_f}$$

Charles' Law (constant n and P): $$\frac{V_i}{T_i} = \frac{V_f}{T_f} \rightarrow V_f = V_i \times \frac{T_f}{T_i}$$

Distribution of Molecular Speeds within a Gas



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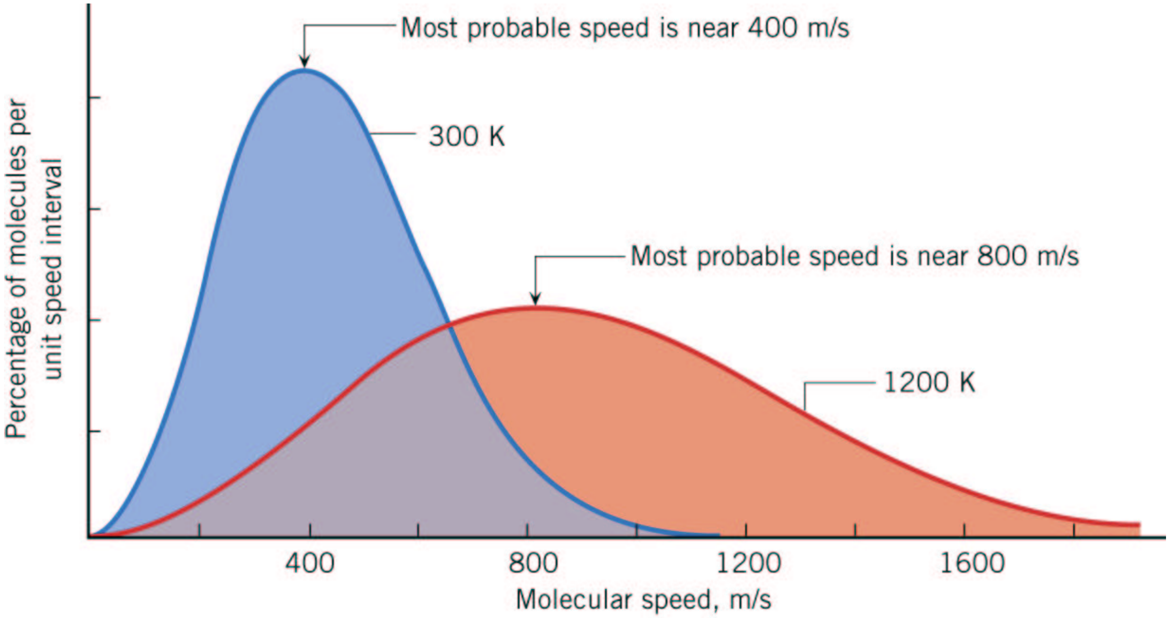

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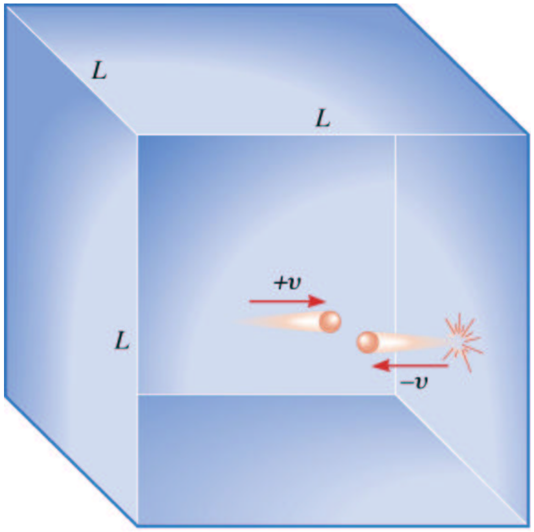
Kinetic Theory



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A particle with a velocity v approaching the right wall has a momentum (mv_i). Once it bounces off the wall, it has a velocity of -v and therefore a momentum ($-mv_f$).

The impulse-momentum theory states:

$$\overline{F}\Delta t = \Delta p = m v_f - m v_i$$

We can rewrite this as:

$$\overline{F} = \frac{\Delta p}{\text{Time between hits}} = \frac{(-mv)-(mv)}{2L/v} = \frac{-mv^2}{L}$$

This is the average force of the wall on the particle.

Kinetic Theory (2)

The previous equation was the average force exerted by the wall on only one particle. The force of the particle on the wall is the same except for a sign change (Newton's 3rd law). We have N such particles in the system all doing this so our equation becomes:

$$F_{\text{total}} = N\left(m\frac{v^2}{L}\right)$$

but any particle may be moving in one of three spatial directions so on average, there are only 1/3 of the total particles hitting the one wall. Also, as seen in a previous slide, there is a distribution of speeds the gas particles have so the velocity one uses here is an average velocity $\bar{v}$.

$$v_{rms} = \sqrt{\overline{v^2}} = \text{root-mean-squared speed}$$

$$F = \frac{N}{3}\left(\frac{m\,v_{\text{rms}}^2}{L}\right)$$

Kinetic Theory (3)

Recall that pressure is Force per Area. Mathematically this is:

$$P = \frac{F}{A} = \frac{F}{L^2} = \frac{N}{3}\left(\frac{m\,v_{\text{rms}}^2}{L}\right)\left(\frac{1}{L^2}\right)$$

$$P = \frac{N}{3}\left(\frac{m\,v_{\text{rms}}^2}{L^3}\right)$$ but L^3 is V

Now multiply top and bottom of the RHS by 2, then take V to the other side and you find:

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\,v_{\text{rms}}^2\right)$$

Kinetic Theory (4)

$$PV = \frac{2}{3} N \left(\frac{1}{2} m v_{\text{rms}}^2 \right)$$

This looks like the ideal gas law:

$$PV = nRT$$

where we can replace n with N / N_A. We also recognize {½ m v^2} as a kinetic energy. Setting both PV equations to be equal we have:

$$\frac{N}{N_A} RT = \frac{2}{3} N \overline{KE}$$

R / N_A is known as Boltzmann's constant and is given the symbol *k*.

$$kT = \frac{2}{3} \overline{KE} \rightarrow \overline{KE} = \frac{3}{2} kT$$

Internal Energy of a Gas

The internal energy of a substance is the sum of all energies the atoms can have. This includes translational energy, potential energy, rotational energy, vibrational energy etc.

For a gas which is monotonic (One atom which can exist by itself naturally. Compare Helium vs. Oxygen. A single atom of He is found in nature but O is normally found paired with another O to form O_2.) let's write out the internal energy:

$$U = N \times \left[(\frac{1}{2} m v^2) + (PE) + (\frac{1}{2} I \omega^2) + (\text{Vibrational Energy}) + \ldots \right]$$

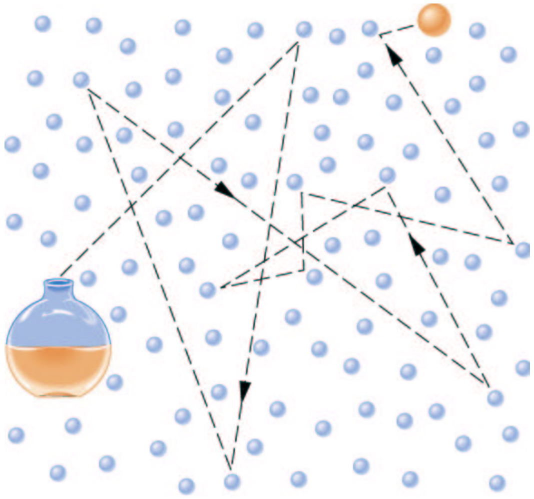
If we cancel everything which is essentially zero we find:

$$U = N\left(\frac{3}{2} k T\right) \text{ but remember, } n = \frac{N}{N_A} \text{ and } \frac{R}{N_A} = k$$

finally we find: $U = \frac{3}{2} n R T$

Diffusion

This is the process by which a gas particle moves from a place of high density (concentration) to a lower density. Let's look at the example of a perfume bottle used in the book.



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If someone opens a bottle of perfume in one corner of the room, we know it takes some time to smell it in the opposite corner. Why since if we assume a perfume particle moves at the average speed an oxygen atom has at room temperature, the perfume should cross a 30 m room in just 75×10^{-3} sec (75 mSec). It takes much longer... Why?